CORE MATHEMATICS (C) UNIT 1 TEST PAPER 5

1. Find the complete set of values of x for which $x^2 \geq 5x + 84$. [4]

2. (i) Given that $4^{3x+1} = 8^{y+1}$, express y in terms of x. [3]

   (ii) Find the value of x for which $4^{3x+1} = 64$. [2]

3. The diagram shows the graph of $y = f(x)$,

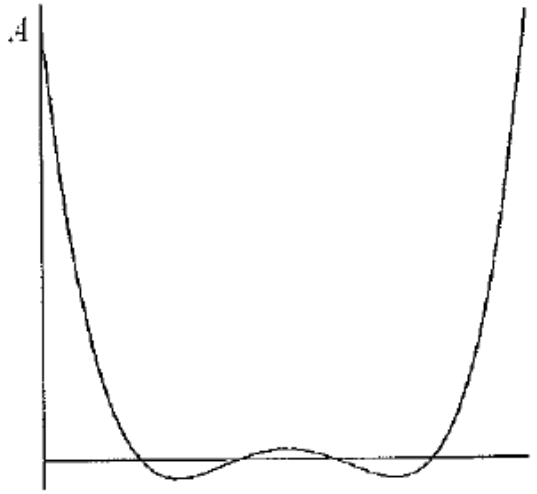
   where $f(x) \equiv (x-1)(x-2)(x-3)(x-4)$.

   The graph crosses the y-axis at A.

   (i) Write down the coordinates of A. [1]

   (ii) Sketch the following graphs, clearly showing the coordinates of the points where they cross the axes:

   (a) $y = -f(x)$, (b) $y = f(x+2)$. [4]



4. (i) Find the integers a and b such that $\dfrac{\sqrt{3}-2}{\sqrt{3}+2} = a\sqrt{3} + b$. [5]

   (ii) Hence or otherwise, solve for x the equation

$$(1-x)\sqrt{3} = 2(x+1).$$ [2]

5. Find $\frac{dy}{dx}$ for each of the following:

(i) $y = (x-4)^2$, (ii) $y = \frac{1}{\sqrt[3]{x}}$, (iii) $y = \frac{x^4 - 2x}{x^3}$. [9]

6. A is the point $(-3, 4)$ and B is the point $(k, -10)$. M is the mid-point of AB.

The straight line through A and B has equation $2x + y + 2 = 0$. Find

(i) the value of k, [2]

(ii) the length of AB, in its simplest surd form. [2]

(iii) the coordinates of M, [2]

(iv) the equation of the line through M perpendicular to AB, in the form $ax + by + c = 0$. [4]

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7. The circle C_1 has equation $x^2 + y^2 - 10x - 12y - 20 = 0$.

   (i) Find the centre and the radius of C_1. [4]

   The circle C_2 has radius 5 cm and centre at (5, 10).

   (ii) Verify that the two circles touch, and find the coordinates of the point which lies on both circles. [4]

   (iii) State the equation of the common tangent to the two circles. [2]

8. In this question, $f(x) \equiv 4x^3 + 10x^2 + 5x$.

   (i) Factorise $f(x)$. [1]

   (ii) Solve the equation $f(x) = 0$, giving any irrational roots as surds in their simplest form. [4]

   A and B are the points on the graph of $y = f(x)$ at which the gradient is 5.

   (iii) Find the x-coordinates of A and B. [5]

9. Given that $x^2 + 18x + 100 \equiv (x + p)^2 + q$,

   (i) find the values of the constants p and q. [3]

   (ii) Deduce that the equation $x^2 + 18x + 100 = 0$ has no real roots. [2]

(iii) Sketch the graph of $y = x^2 + 18x + 100$. [2]

(iv) If the equation $x^2 + 18x + 100 = t$ has at least one real root, find the set of possible values of t. [2]

(v) State the value of t for which $x^2 + 18x + 100 = t$ has a repeated root, and find this root. [3]

CORE MATHS 1 (C) TEST PAPER 5 : ANSWERS AND MARK SCHEME

1. $(x + 7)(x - 12) \geq 0$ $x \leq -7, x \geq 12$ M1 A1 A1 A1 4

2. (i) $2^{6x+2} = 2^{3y+3}$ $3y = 6x - 1$ $y = 2x - 1/3$ M1 A1 A1
   (ii) Here $y = 1$, so $x = 2/3$ M1 A1 5

3. (i) A is (0, 24) (ii) Graph (a) reflected in x-axis, B1 B2
   (b) translated –2 units in x direction: through (–1, 0), (0, 0), (1, 0), (2, 0) B2 5

4. (i) $\sqrt{3} - 2 = (a\sqrt{3} + b)(\sqrt{3} + 2) = 3a + 2b + (2a + b)\sqrt{3}$ M1 A1
   $2a + b = 1$, $3a + 2b = -2$ $a = 4$, $b = -7$ M1 A1 A1
   (ii) $(\sqrt{3} + 2)x = \sqrt{3} - 2$, so $x = 4\sqrt{3} - 7$ M1 A1 7

5. (i) $y = x^2 - 8x + 16$ $dy/dx = 2x - 8$ B1 M1 A1
   (ii) $y = x^{-1/3}$ $dy/dx = -\frac{1}{3}x^{-4/3}$ B1 M1 A1
   (iii) $y = x - 2x^{-2}$ $dy/dx = 1 + 4/x^3$ B1 M1 A1 9

6. (i) $2k - 10 + 2 = 0$ $k = 4$ M1 A1
   (ii) $AB^2 = 7^2 + 14^2$ $AB = 7\sqrt{5}$ M1 A1
   (iii) $M = (\frac{1}{2}, -3)$ B1 B1

(iv) Gradient of $AB = -2$ Gradient of perp. $= \frac{1}{2}$ B1 B1

$y + 3 = \frac{1}{2}(x - \frac{1}{2})$ $2x - 4y - 13 = 0$ M1 A1 10

7. (i) $(x-5)^2 + (y-6)^2 = 81$ Centre (5, 6), radius 9 M1 A1 A1 A1

(ii) $x = 5$ is common diameter and (5, 15) is on both circles, so they touch there B1 M2 A1

(iii) Common tangent is $y = 15$ B2 10

8. (i) $f(x) = x(4x^2 + 10x + 5)$ B1

(ii) $x = 0$ or $x = \dfrac{-10 \pm \sqrt{20}}{8} = \dfrac{-5 \pm \sqrt{5}}{4}$ B1 M1 A1 A1

(iii) $12x^2 + 20x + 5 = 5$ $4x(3x + 5) = 0$ $x = 0, x = -5/3$ M1 A1 M1 A1 A1 10

9. (i) $p = 9, q = 19$ M1 A1 A1

(ii) $(x + 9)^2 + 19 > 0$ for all real x B2

(iii) Quadratic graph with minimum at (–9, 19) B2

(iv) $t \geq 19$ M1 A1

(v) $t = 19$; then $x = -9$ M1 A1 A1 12